Anisotropy and Interference in Wave Transport: An Analytic Theory

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A theory is presented which incorporates the effect of dielectric anisotropy in random multiple scattering media. It predicts anisotropic diffusion, and a deflection of the diffuse energy flow in anisotropic slabs in the direction parallel to the slab. The transmittance integrated over all incoming and outgoing directions scales with the transport mean free path along the surface normal. The escape function in anisotropic dielectrics is no longer bell shaped. In this model anisotropy facilitates Anderson localization.

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The effects of anisotropy on transport of waves are currently studied in a variety of fields such as electronics, seismology, biology, and optics [1–13]. An understanding of the effect of anisotropy on wave transport is crucial for acquiring information about the structure of scattering materials. Both ballistic propagation and multiple scattering of seismic waves are affected by the anisotropy of Earth's crust [1–4]. Furthermore, anisotropy is very important in diffuse optical tomography, where anisotropic diffusion is observed in biological tissue such as skin, teeth, muscle, and bone [5,6]. Interference effects in wave transport are also strongly influenced by anisotropy, e.g., enhanced backscattering of light in semiconductors or liquid crystals [7–11]. Ultimately interference can lead to the extreme case of Anderson localization, a situation in which the radiance is confined by interference effects in a random scattering medium. Strong anisotropy could effectively reduce a 3-dimensional medium to a quasi 1- or 2-dimensional medium where Anderson localization might be much easier to observe [12,13].

In this Letter, we explain the effect of anisotropy on the diffusion of scalar waves through randomly distributed scatterers in an anisotropic host medium. We start with amplitude properties in homogeneous, anisotropic host media and then we add scatterers. From the amplitude properties we derive diffuse energy density transport. Whereas in isotropic media the diffusion constant is known to be a product of a transport mean free path and a transport velocity, $D = vl/3$, we find a diffusion tensor in which the transport mean free path and transport velocity turn out to be vectors. A key observable of illuminated random media is the escape function [14], which describes the angular dependence of the radiance that escapes the medium. We show that the escape function, which has a universal appearance for isotropic media, strongly depends on the anisotropy. Finally we investigate the effect of anisotropy on wave interference and we derive a Ioffe-Regel criterion for Anderson localization in anisotropic media.

Our starting point is a homogeneous, anisotropic scalar wave equation,

$$\left[\boldsymbol{\nabla}\cdot\mathsf{A}\cdot\boldsymbol{\nabla} - \frac{1}{c_i^2}\frac{\partial^2}{\partial t^2}\right]\psi(t,\boldsymbol{x}) = 0. \quad (1)$$

The energy density $\mathcal{H}$ associated with ψ is

$$\mathcal{H} = \frac{1}{2}\left[\frac{1}{c_i^2}\frac{\partial\psi^*}{\partial t}\frac{\partial\psi}{\partial t} + \boldsymbol{\nabla}\psi^*\cdot\mathsf{A}\cdot\boldsymbol{\nabla}\psi + \text{c.c.}\right]. \quad (2)$$

We map (2) on the energy density of electromagnetic waves in a homogeneous, anisotropic dielectric, with permittivity tensor ε and permeability scalar μ, both real valued. We identify the dimensionless anisotropy tensor $\mathsf{A} \equiv 3\varepsilon^{-1}/\mathrm{Tr}(\varepsilon^{-1})$ and isotropic velocity $c_i^2 = \mathrm{Tr}(\varepsilon^{-1})/(3\mu)$. A similar mapping can be found in [15], but with the electric and magnetic fields interchanged. In our description of the electromagnetic field by a scalar potential the transverse polarization of the electromagnetic fields is essentially lost. The wave character is retained, and we expect our scalar theory to be able to describe energy transport in anisotropic media of electromagnetic waves which have fully scrambled polarization.

In a homogeneous anisotropic space both the phase velocity v_p (a scalar) and the group velocity $\boldsymbol{v}_g$ (a vector) are real quantities and depend on the direction of the wave vector $\boldsymbol{e}_k$. They are given by

$$v_p(\boldsymbol{k}) \equiv \frac{\omega(\boldsymbol{k})}{|\boldsymbol{k}|} = c_i\sqrt{\boldsymbol{e}_k\cdot\mathsf{A}\cdot\boldsymbol{e}_k}, \quad (3)$$

$$\boldsymbol{v}_g(\boldsymbol{k}) \equiv \frac{\partial\omega(\boldsymbol{k})}{\partial\boldsymbol{k}} = \frac{c_i^2}{v_p(\boldsymbol{e}_k)}\mathsf{A}\cdot\boldsymbol{e}_k, \quad (4)$$

where $\omega(\boldsymbol{k})$ is the dispersion relation in the homogeneous anisotropic medium. Only along the principal axes $\boldsymbol{e}_a$ of the dielectric tensor we have $\boldsymbol{v}_g(\boldsymbol{e}_a) = v_p(\boldsymbol{e}_a)\boldsymbol{e}_a$. We define a homogeneous but anisotropic refractive index m for the medium without scatterers by

$$m(\boldsymbol{e}_k) \equiv \frac{c_0}{c_i}\frac{1}{\sqrt{\boldsymbol{e}_k\cdot\mathsf{A}\cdot\boldsymbol{e}_k}}, \quad (5)$$

with c_0 the velocity of light in vacuum. Refractive index (5) is a nontrivial generalization of $m = c_0/c_i$ valid for isotropic media.

The scatterers are introduced by means of a frequency ω dependent scattering potential

$$V_\omega(\boldsymbol{x}, \boldsymbol{x}') \equiv -\frac{\omega^2}{c_i^2}\left[\frac{\mu(\boldsymbol{x})}{\mu} - 1\right]\delta^3(\boldsymbol{x} - \boldsymbol{x}'). \tag{6}$$

If we were to define scatterers as inhomogeneities in the permittivity, then we would introduce unwanted nonlocal effects. The scalar Helmholtz equation for the amplitude Green function G in the presence of scatterers is

$$\left[\boldsymbol{\nabla}\cdot\mathsf{A}\cdot\boldsymbol{\nabla} + \frac{\omega^2}{c_i^2}\right]G_\omega(\boldsymbol{x},\boldsymbol{x}_0) = \delta^3(\boldsymbol{x}-\boldsymbol{x}_0) + \int d^3x_1 V_\omega(\boldsymbol{x},\boldsymbol{x}_1)G_\omega(\boldsymbol{x}_1,\boldsymbol{x}_0). \tag{7}$$

A single elastic scatterer placed in an anisotropic medium gives rise to scattering and extinction cross sections σ_s and σ_e, for which we establish the optical theorem $\sigma_s = \sigma_e$. The cross sections generalized to anisotropic media we found are in terms of the single scatterer T matrix $T_\omega(\boldsymbol{e_k}, \boldsymbol{e}_{\boldsymbol{k}_1})$ given by

$$\sigma_s(\boldsymbol{e_k}) \equiv \frac{\langle T_\omega(\boldsymbol{e_k}, \boldsymbol{e}_{\boldsymbol{k}_1})T^*_\omega(\boldsymbol{e_k}, \boldsymbol{e}_{\boldsymbol{k}_1})\rangle_{\boldsymbol{e}_{\boldsymbol{k}_1}}}{4\pi\sqrt{\det\mathsf{A}}}, \tag{8}$$

$$\sigma_e(\boldsymbol{e_k}) \equiv -\frac{c_i \mathrm{Im}(T_\omega(\boldsymbol{e_k}, \boldsymbol{e_k}))}{\omega}. \tag{9}$$

where we suppress the dependence on ω. New in the scattering cross section (8) is anisotropy of the surrounding medium, which enters through the average $\langle\ldots\rangle$ over the anisotropic surface at constant frequency,

$$\langle\ldots\rangle_{\boldsymbol{e_k}} \equiv \int \frac{d^2e_k}{4\pi}\frac{\ldots}{\sqrt{(\boldsymbol{e_k}\cdot\mathsf{A}\cdot\boldsymbol{e_k})^3\det\mathsf{A}^{-1}}}, \tag{10}$$

a special case of a three dimensional integral over the dispersion relation for which we can use a coordinate transformation to calculate $\langle 1\rangle_{\boldsymbol{e_k}} = 1$ and $\langle \boldsymbol{e_k}\boldsymbol{e_k}\rangle_{\boldsymbol{e_k}} = \mathsf{A}^{-1}/3$. Furthermore, the momentum transfer cross section σ is in anisotropic media found to be

$$\sigma(\boldsymbol{e_k}) \equiv \frac{1}{3}\boldsymbol{v}_g(\boldsymbol{e_k})\cdot\langle\boldsymbol{v}_g(\boldsymbol{e}_{\boldsymbol{k}_2})\boldsymbol{v}_g(\boldsymbol{e}_{\boldsymbol{k}_2})\rangle^{-1}_{\boldsymbol{e}_{\boldsymbol{k}_2}} \cdot\int d^2e_{k_1}\frac{d\sigma_s(\boldsymbol{e_k}, \boldsymbol{e}_{\boldsymbol{k}_1})}{d^2e_{k_1}}\{\boldsymbol{v}_g(\boldsymbol{e_k}) - \boldsymbol{v}_g(\boldsymbol{e}_{\boldsymbol{k}_1})\}, \tag{11}$$

where $\boldsymbol{v}_g(\boldsymbol{e_k})\cdot\langle 3\boldsymbol{v}_g(\boldsymbol{e}_{\boldsymbol{k}_2})\boldsymbol{v}_g(\boldsymbol{e}_{\boldsymbol{k}_2})\rangle^{-1}_{\boldsymbol{e}_{\boldsymbol{k}_2}}\cdot[\boldsymbol{v}_g(\boldsymbol{e_k}) - \boldsymbol{v}_g(\boldsymbol{e}_{\boldsymbol{k}_1})]$ is the well known factor $1 - \cos\theta \equiv 1 - \boldsymbol{e_k}\cdot\boldsymbol{e}_{\boldsymbol{k}_1}$ generalized to anisotropic media [15–18]. In the independent scattering limit for scatterer density n we introduce the scattering mean free path $l_s(\boldsymbol{e_k}) \equiv |\boldsymbol{v}_g(\boldsymbol{e_k})|/[c_i n\sigma_s(\boldsymbol{e_k})]$.

The conserved quantity in multiple scattering of light is energy, which is related to the radiance. We want to set up a transport equation for the energy density, or for the radiance, which is conserved along a flow line [19]. For the anisotropic medium with isotropic scatterers we can derive a Bethe-Salpeter equation for the ensemble averaged product of amplitude Green functions $\langle\langle G^*G\rangle\rangle$. The Bethe-Salpeter equation is a generalized transport equation in the sense that it contains all interference effects [15]. We introduce the specific intensity or radiance per unit frequency band by approximating the dispersion relation with the one for the anisotropic medium without scatterers. We integrate the Bethe-Salpeter equation over the wave vector magnitude to obtain a radiative transfer equation for the radiance.

We obtain an anisotropic diffusion equation for the total energy density if we expand the radiance self-consistently in terms of the radiative energy density and flux [15,16,19–21]. The first two moments of the radiative transfer equation give rise to a continuity equation and a Fick law relating the total energy density and flux.

The diffusion constant we obtain is a second rank symmetric tensor, and is also obtainable from the Kubo formalism [9,18,22,23]. We find

$$\mathsf{D} = \frac{\tau\langle\boldsymbol{v}_g(\boldsymbol{e_k})\boldsymbol{v}_g(\boldsymbol{e_k})\rangle_{\boldsymbol{e_k}}}{1+\delta} = \frac{1}{3}\frac{\tau c_i^2}{1+\delta}\mathsf{A}. \tag{12}$$

In (12) we have in order of appearance a mean free time τ, the group velocity $\boldsymbol{v}_g$ from (4), and a δ which is related to the energy density temporarily stored in the elastic scatterers [15,24]. In the independent scattering limit for scatterer density n, the average mean free time τ is

$$c_i\tau \equiv \frac{1}{n\langle\sigma(\boldsymbol{e_k})\rangle_{\boldsymbol{e_k}}}. \tag{13}$$

In experiments the scattering medium is always bounded. Consider a semi-infinite scattering medium with the boundary plane through some $\boldsymbol{x}$ with unit surface normal $\boldsymbol{n}_\perp$. At $\boldsymbol{x}$ all energy density flux components headed into the scattering medium should add up to zero, as there is no diffuse energy density w present outside the scattering medium. If we set this requirement we find the boundary condition for diffusion in media with anisotropic dispersion

$$0 \equiv \tfrac{1}{4}\boldsymbol{n}_\perp\cdot\boldsymbol{v}(\boldsymbol{n}_\perp)w(t,\boldsymbol{x}) - \tfrac{1}{2}\boldsymbol{n}_\perp\cdot\mathsf{D}\cdot\boldsymbol{\nabla}w(t,\boldsymbol{x}), \tag{14}$$

where $\boldsymbol{v}(\boldsymbol{n}_\perp)$ the energy transport velocity. We can formulate boundary condition (14) for boundaries of arbitrary orientation $\boldsymbol{n}_\perp$, and we identify the transport velocity $\boldsymbol{v}$ to be

$$\boldsymbol{v}(\boldsymbol{e_k}) = \frac{\boldsymbol{v}_g(\boldsymbol{e_k})}{1+\delta}, \tag{15}$$

and note that in (14) $\boldsymbol{v}(\boldsymbol{n}_\perp)$ points along $\mathsf{A}\cdot\boldsymbol{n}_\perp$. The transport mean free path vector can now be defined by

$$\boldsymbol{l}(\boldsymbol{e_k}) \equiv \frac{3\boldsymbol{e_k}\cdot \mathsf{D}}{\boldsymbol{e_k}\cdot \boldsymbol{v}(\boldsymbol{e_k})} = \tau \boldsymbol{v}_g(\boldsymbol{e_k}). \tag{16}$$

Equations (12) and (14)–(16) lead to the equivalent boundary condition $w(t, \boldsymbol{x}) = 2\boldsymbol{l}(\boldsymbol{n}_\perp)\cdot \nabla w(t, \boldsymbol{x})/3$, so $\boldsymbol{l}(\boldsymbol{n}_\perp)$ is for a given boundary directed along $\mathsf{A}\cdot \boldsymbol{n}_\perp$.

The diffusion tensor (12) becomes an average (10) over the product of (15) and (16),

$$\mathsf{D} = \langle \boldsymbol{v}(\boldsymbol{e_k})\boldsymbol{l}(\boldsymbol{e_k})\rangle_{\boldsymbol{e_k}} = \sum_{i=1}^{3} \frac{1}{3} \boldsymbol{v}(\boldsymbol{e}_i)\boldsymbol{l}(\boldsymbol{e}_i)\cdot \boldsymbol{e}_i\boldsymbol{e}_i, \tag{17}$$

with $\{\boldsymbol{e}_i\}$ the three principal axes of the anisotropy, and in isotropic media we obtain $\mathsf{D} = |\boldsymbol{v}||\boldsymbol{l}|\mathbb{1}/3$.

The escape function governs the distribution of escaping radiance over angles. For semi-infinite media we calculate

$$K(\boldsymbol{e_k}, \boldsymbol{n}_\perp) = \frac{3}{2N_{\boldsymbol{n}_\perp}} \left| \frac{\boldsymbol{v}_g(\boldsymbol{e_k})}{c_i} \right|^2 u(\boldsymbol{e_k}, \boldsymbol{n}_\perp)[\tau_e(\boldsymbol{n}_\perp) + u(\boldsymbol{e_k}, \boldsymbol{n}_\perp)], \tag{18}$$

where $N_{\boldsymbol{n}_\perp}$ is defined by $\langle K(\boldsymbol{e_k}, \boldsymbol{n}_\perp)\rangle_{\boldsymbol{e_k}} \equiv 1$ with $\boldsymbol{e_k}$ limited to all outgoing flux components, $\boldsymbol{n}_\perp \cdot \boldsymbol{v}_g(\boldsymbol{e_k}) \geq 0$, τ_e is the extrapolation ratio

$$\tau_e(\boldsymbol{n}_\perp) \equiv \frac{2\boldsymbol{l}(\boldsymbol{n}_\perp)\cdot \boldsymbol{n}_\perp}{3l_s(\boldsymbol{n}_\perp)}, \tag{19}$$

which for isotropic scatterers becomes $\tau_e = 2/3$, and u is the generalization of the $\cos\theta = \boldsymbol{n}_\perp \cdot \boldsymbol{e_k}$ known from the isotropic medium [14], u is given by

$$u(\boldsymbol{e_k}, \boldsymbol{n}_\perp) = \sqrt{1 - \left(\frac{m(\boldsymbol{e_k})}{m(\boldsymbol{e}_{k_\parallel})} \boldsymbol{e}_{k_\parallel} \cdot \boldsymbol{e_k}\right)^2}, \tag{20}$$

where the unit vector $\boldsymbol{e}_{k_\parallel}$ parallel to the boundary is the direction along the components of $\boldsymbol{k}$ perpendicular to $\boldsymbol{n}_\perp$,

$$\boldsymbol{e}_{k_\parallel} \equiv \frac{[\mathbb{1} - \boldsymbol{n}_\perp\boldsymbol{n}_\perp]\cdot \boldsymbol{e_k}}{\sqrt{1 - (\boldsymbol{e_k}\cdot \boldsymbol{n}_\perp)^2}}. \tag{21}$$

If we have on both sides of the boundary an isotropic medium, then $m(\boldsymbol{e_k}) = m(\boldsymbol{e}_{k_\parallel})$, and u reduces to $\cos\theta$,

$$u_{\rm iso}(\boldsymbol{e_k}, \boldsymbol{n}_\perp) = \sqrt{1 - (\boldsymbol{e_k}\cdot \boldsymbol{e}_{k_\parallel})^2} = \sqrt{(\boldsymbol{e_k}\cdot \boldsymbol{n}_\perp)^2}. \tag{22}$$

We calculate escape function (18) for realistic values of the anisotropic permittivity and isotropic scatterers, see Fig. 1. In contrast to the universal appearance of the escape function in isotropic media [14], we predict nonuniversal behavior. The shift of the position of the maximum and the appearance of a second maximum are related to the fact that $|\boldsymbol{l}|$ has its minimum where $|\boldsymbol{v}_g|$ is minimal. For anisotropy parallel to the surface we predict two maxima. The energy density current is deflected in the direction perpendicular to the surface normal. We note that it is only possible to perfectly index match an anisotropic medium with isotropic scatterers to an outside world if the anisotropy axes are parallel or perpendicular to the boundary. In general the escape function acquires an imaginary part when $u^2 < 0$, implying internal reflection.

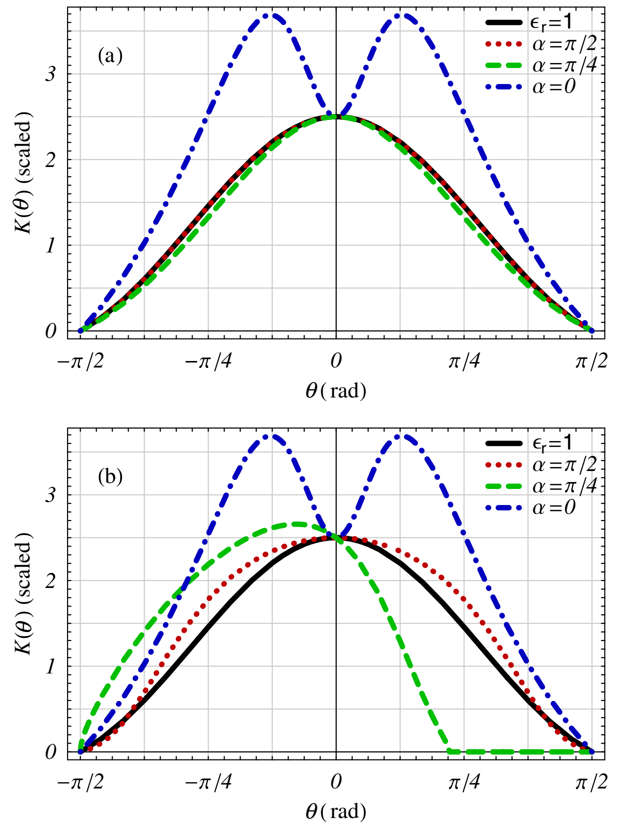

FIG. 1 (color online). Escape functions $aK(\theta)$ for a semi-infinite medium with isotropic point scatterers, and a plane source inside. The anisotropic dielectric tensor $\varepsilon_r = \mathbb{1} + 3\boldsymbol{dd}$, with unit vector $\boldsymbol{d}$ the director of the anisotropy. Angle θ of wave vector $\boldsymbol{e_k}$ and outward pointing surface normal $\boldsymbol{n}_\perp$ is defined by $\cos\theta \equiv \boldsymbol{e_k}\cdot \boldsymbol{n}_\perp$. The escape function is plotted for several director orientations α defined by $\cos\alpha \equiv \boldsymbol{d}\cdot \boldsymbol{n}_\perp$. Scale factor $a \equiv 2\pi K_{\varepsilon_r=1}(0)/K_{\varepsilon_r}(0)$ makes all $\theta = 0$ values coincide. In (a) $\boldsymbol{e_k}$ is in the plane with normal $\boldsymbol{n}_\perp \times (\boldsymbol{d}\times \boldsymbol{n}_\perp)$, and in (b) in the plane with normal $\boldsymbol{d}\times \boldsymbol{n}_\perp$. We see an anisotropic bell shape for $\alpha = \pi/2$. When $\alpha = \pi/4$ perfect index matching is impossible and we get internal reflection for $\theta > \pi/4$. When $\alpha = 0$ we see two maxima, with symmetry around $\boldsymbol{d}$.

In slab geometries we have two boundaries. With $\boldsymbol{n}_\perp$ the outward pointing boundary surface normal, we calculate the angle resolved transmission function $T(\boldsymbol{e}_i, \boldsymbol{e}_s, \boldsymbol{n}_\perp)$. For slabs of thickness $L \gg l_s(\boldsymbol{n}_\perp)$ only diffuse light is transmitted and we find

$$T(\boldsymbol{e}_i, \boldsymbol{e}_s, \boldsymbol{n}_\perp) = \frac{4}{3} \frac{l_\perp(\boldsymbol{n}_\perp)K(\boldsymbol{e}_i, \boldsymbol{n}_\perp)K(\boldsymbol{e}_s, \boldsymbol{n}_\perp)}{L + \frac{4}{3}l_\perp(\boldsymbol{n}_\perp)}, \tag{23}$$

where the subscripts i and s stand for incoming wave and scattered wave, respectively. Equation (23) has the expected plain form of the isotropic medium result [14], containing a transport mean free path of which we have shown that in general it is a vector component. If we integrate (23) over all incoming and outgoing directions the escape functions K disappear, and we find that the result is $T(\boldsymbol{n}_\perp) = 4l_\perp(\boldsymbol{n}_\perp)/(3L)$, proportional to the transport length, and all $\boldsymbol{v}_g$ dependence has disappeared.

We can generalize the Ioffe-Regel criterion for infinite isotropic media to infinite anisotropic media. Following [22], we improve the Boltzmann approximation to the irreducible Bethe-Salpeter vertex by adding the maximally crossed correction

$$U = n|T|^2 + \frac{4\pi}{c_i \tau^2} \frac{1}{(\boldsymbol{p}+\boldsymbol{p}')\cdot \mathsf{D} \cdot (\boldsymbol{p}+\boldsymbol{p}')}. \tag{24}$$

The first term describes isotropic point scatterers in the Boltzmann limit $n|T|^2$, self-consistency imposes that D be the diffusion constant including the maximally crossed correction. Together with a wave vector cutoff given by $\boldsymbol{k}_{\max} = \beta/\boldsymbol{e}_k \cdot \boldsymbol{l}(\boldsymbol{e}_k)$, with parameter $\beta = \pi/6$ to recover the isotropic Ioffe-Regel criterion $kl = 1$, this leads to

$$\mathsf{D}^{-1} = \mathsf{D}_B^{-1} + \frac{1}{\det\mathsf{A}\langle \boldsymbol{k}\cdot \boldsymbol{l}(\boldsymbol{e}_k)\rangle^2_{\boldsymbol{e}_k}} \mathsf{D}^{-1}, \tag{25}$$

where D_B is given by (12). It is generally accepted that kl quantizes scattering strength, of which $\langle \boldsymbol{k}\cdot \boldsymbol{l}(\boldsymbol{e}_k)\rangle_{\boldsymbol{e}_k}$ is the logical generalization to anisotropic media. The transition to Anderson localization is at

$$\langle \boldsymbol{k}\cdot \boldsymbol{l}(\boldsymbol{e}_k)\rangle_{\boldsymbol{e}_k} = \sqrt{\frac{1}{\det\mathsf{A}}}, \tag{26}$$

and is affected by the anisotropy in the dielectric tensor through $\det\mathsf{A}$. For elastic scatterers we can check the scaling of the left-hand side of (26) through Eqs. (8) and (9), which imply that the cross sections are proportional to $\sqrt{\det\mathsf{A}}$, making the transport mean free path inversely proportional to $\sqrt{\det\mathsf{A}}$. In terms of the positive eigenvalues ε_i of the dielectric tensor ε the right-hand side of (26) reads $\sqrt{(1/3)^3\varepsilon_1\varepsilon_2\varepsilon_3(1/\varepsilon_1 + 1/\varepsilon_2 + 1/\varepsilon_3)^3}$, and achieves a minimum for isotropic media. Therefore, while keeping the inhomogeneity (6) of the medium fixed, we can obtain Anderson localization in anisotropic media for higher values of $\langle \boldsymbol{k}\cdot \boldsymbol{l}(\boldsymbol{e}_k)\rangle_{\boldsymbol{e}_k}$. For example, in dielectrics with $\varepsilon_1 = \varepsilon_2 = (2/11)\varepsilon_3$ the transition occurs at 16/11, which is larger than the isotropic value 1. This result could have been expected from the wave equation (1), where an axis with high dielectric constant tends to reduce the relative importance of that spatial dimension.

In conclusion, we found that the vector transport mean free path and the vector transport velocity depend on the surface normal and the principal axes of the anisotropy. The escape function in anisotropic dielectrics is no longer the well-known expression $K(\theta) = \cos\theta[1 + (3/2)\cos\theta]$, which is universal in isotropic media and often applied to anisotropic media, but we find that the anisotropic host medium can deflect the diffuse energy density current. In a slab geometry the transmittance integrated over all incoming and outgoing directions is proportional to the transport length along the surface normal. Our model, with homogeneous anisotropy, confirms a more general picture that anisotropic dielectrics are advantageous for Anderson localization.

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